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Prospective Primary School Teachers' Work in Continuous Online Assessments in the Course of Didactics of Mathematics

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Abstract: This study aimed to examine students' work in online assessments to gain more understanding for designing continuous assessments in a blended learning environment for prospective primary school teachers. The study took place during emergency remote teaching due to the COVID-19 pandemic. Course work for prospective primary school teachers in the didactics of mathematics course included continuous, obligatory, non-graded, online assessments. We performed a qualitative content analysis of their answers. Students' work was examined regarding the content knowledge and requirements in the questions, based on the categories of Subject Matter Knowledge and Mathematical Assessment Task Hierarchy taxonomy. The results showed that students' study approach was strategic, relying heavily on peer support. Their work differed concerning the content and requirements of the questions. Students were more engaged in questions that required creating examples, discussing definitions and properties, and solving contextual problems. Questions related to horizon content knowledge were most challenging for students. We discussed how the results of our study could affect the design of continuous assessment in a blended learning environment for prospective primary school teachers.

INTRODUCTION

Prospective primary school teachers in Croatia are educated in different subject areas, physical sciences, mathematics, computer science, arts and humanities, psychology, and other social sciences. They have university-level service courses and didactics courses related to each primary level subject, including mathematics. Students enter university with different secondary education profiles and interests and struggle with the composite nature of their studies, with mathematics courses. There might be different reasons for their difficulties and failure in mathematics, math anxiety, overladen syllabus, weak prior knowledge, and inappropriate study approach. Procrastination and postponing work just before the examination, a common practice among the student population, seemed a particular issue.

Due to the COVID-19 global pandemic, education on all levels was disrupted and abruptly shifted to emergency remote teaching (ERT). There were many challenges for educational institutions and their stakeholders, for example, the dependence on technology and issues with socio-economic equality, compatibility of educational content and achievements with channels for remote teaching and learning, supervision and adaptation of the assessments in a digital environment, increased teachers and students' workload, teachers' professional and digital competencies, students' digital competencies and their engagement (Aldon et al., 2021; Tanujaya et al., 2021; Tay et al., 2021). When digital resources are integrated in a meaningful way, they can enhance the environment by providing alternative, multiple and interactive representations, accessibility of resources, communications in all channels, and increase students' self-efficacy through differentiation, self-paced opportunities, and personalized feedback (Attard & Holmes, 2020; Borba et al., 2016; Jamil et al., 2022; Tanujaya et al., 2021; Wang, 2021). Our intention to use the knowledge gained in these unprecedented circumstances to improve regular and online education motivated this study.

The concern about students' activity in mathematics courses increased during the COVID-19 lockdown. Our lecture-based course in didactics of mathematics went completely remote to the digital environment. To compensate, as a part of the course work, we engaged students in continuous, obligatory, non-graded, online assessments with individual feedback on their work. Research indicates that frequent formative assessment could engage students in continuous work and move them from learning for examination to active learning. Korhonen et al. (2015) reported that constant workload eased the burden before the final exam, contributed to students' understanding, and prompted small group collaboration. Cusi and Telloni (2019) found that university students valued the effectiveness of a designed individualized online path with feedback to support their learning. Continuous work in the form of non-graded assignments and writing in mathematics can contribute to learning and teaching mathematics (Flesher, 2003; Kuzle, 2013). Building on this novel experience of organizing course work, we questioned if such assignments synchronized with course content could complement the lectures into a blended learning environment, as a combination of online and face-to-face activities (Borba et al., 2016; Dio, 2022; Tanujaya et al., 2021).

The purpose of this study was to analyse students' work in online assessments implemented during COVID-19 remote teaching. The results would provide understanding for the future design of continuous assessments in a blended learning environment for university mathematics education of prospective teachers.

The paper has five sections. The literature review contains research about continuous assessment and theoretical constructs used in the analysis of students' work, and it ends with stating research questions. Regarding the methodology section, we describe the context of the study, the study instrument supported with the theoretical constructs and the data analysis process. The section with results is organized according to the stated research questions. In the discussion section, we connect and lead the results toward the study goal, that is, the issue of designing online, continuous assessments for prospective teachers. We also discuss limitations, implications and ideas for further research. The final section is the conclusion.

LITERATURE REVIEW

Continuous assessment and active learning

Assessment is an inevitable part of education. Including different aspects of the notion addressed in educational research, Joughin wrote that assessment is "to make judgements about students' work, inferring from this what they can do in the assessed domain, and thus what they know, value, or are capable of doing" (2009, p. 16). Goos (2014) added the intention of using assessment results to plan further educational actions. There are multiple assessment purposes, but the authors emphasised evaluating students' knowledge and supporting their learning (Goos, 2014; Hernández, 2012; Hughes, 2008; Trotter, 2006). The effect of assessment on student learning was extensively researched yet unclear (Joughin, 2009; Rust, 2002). Students' approach to learning depends on the mode of assessment, but also the teaching pedagogy, learning environment, attractiveness and relevance of the course content, personal attitudes and goals, and others (Darlington, 2019; Joughin, 2009).

In mathematics education, approaches to learning are described with students' engagement and achievement goals (Dahl, 2017; Darlington, 2019; Jukić Matić et al., 2013). A surface approach to learning is a low-demanding approach focused on avoiding failure. Students memorise the whole material to perform a particular task without understanding. Students who approach the content intending to understand, actively engage in the study and make connections between material have a deep learning approach. A strategic approach to learning is using the least demanding study organization to achieve the best possible examination result. It is important to motivate students to attempt to understand and connect mathematical content, rather than rely on reproducing or memorising facts and even solutions as a part of a surface or surface-strategic approach (Darlington, 2019).

The continuous, formative, learner-oriented and criterion-referenced assessment had a positive impact on students learning (Hernández, 2012; Nair & Pillay, 2004; Patterson et al., 2020; Rust, 2002; Shorter & Young, 2011; Trotter, 2006). Some arguments for effective continuous assessment follow:

- Frequent assignments optimise workload and encourage regular work.
- Smaller-scope and relevant (real-life) tasks raise interest and engagement.
- Prompt, constructive and criterion-related feedbacks are useful.

In the context of our study, a continuous assessment was a part of course work to engage students to regularly reflect on course material and advance based on the teachers' feedback about their productions (Shorter & Young, 2011).

Prospective teachers work on assessment items

The term work, following Joughin's definition, referred to students' productions in assessment items, from which we made inferences about their knowledge and skills. We evaluated students' work from two perspectives: the category of the knowledge at stake and the category of the requirements in the task.

There is a general agreement that teachers' knowledge should be multidimensional; the theoretical and empirical research differentiate, among others, the content and pedagogical aspects of teachers' knowledge (Schwarz & Kaiser, 2019). In this study, we used the mathematics knowledge for teaching framework proposed by Ball et al. (2008). They described the knowledge required for teaching with several categories distributed among subject matter knowledge and pedagogical content knowledge. Pedagogical content knowledge (PCK) includes knowledge about content and students and knowledge about content and teaching, which refer to peculiarities of learning and instruction for particular mathematical content in a particular educational setting, and knowledge of content and curriculum. Subject matter knowledge (SMK) includes common content knowledge (CCK) as knowledge and activities used in any non-educational context, including formal mathematics, specialized content knowledge (SCK) as knowledge and activities used in teaching mathematics, and horizon content knowledge (HCK) as awareness and ability to vertically correlate mathematics knowledge and activities or observe school content from an advanced point of view. Categories of SMK focus on the work, choices and actions grounded in mathematics whereas PCK relates to pedagogically oriented ideas and choices in teaching.

Teachers' activities related to SCK are presenting and discussing mathematical ideas, examining, selecting and connecting representations, constructing and modifying examples and problems, and interpreting and justifying solutions (Ball et al., 2008; Hill et al., 2004). Mathematical courses for prospective teachers should incorporate assessment items which promote mathematical work relevant for teaching, related not only to CCK but also to HCK, and in particular to SCK (Patterson et al., 2020; Selling et al., 2016).

Questions about the same content can be formulated with different requirements for students' skills. We used a modification of Bloom's taxonomy for structuring assessment tasks to categorise questions based on their requirements (Smith et al., 1996). The Mathematical Assessment Task Hierarchy (MATH) taxonomy consists of eight categories organised into three groups (Table 1). Each category has descriptors of skills and activities required for solving tasks. The categories are not hierarchical and do not relate to the complexity of the mathematical content or the subjective difficulty a student might face with a given task, but the focus is on the mathematical demand of the task (Darlington, 2014). The MATH taxonomy was used to compare examinations (Darlington, 2014; Kinnear et al., 2020), and analyse course material (Bennie, 2005), and researchers suggest it could be used when developing assessments and curricula.

Research questions

It is problematic to assume that continuous assessment promotes a deep approach to learning, but the cost of implementing continuous assessment in a blended learning environment is worth the potential benefits for students learning (Attard & Holmes, 2020; Trotter, 2006). Evaluating and redesigning assessments regarding students' work can contribute to a more balanced, effective and meaningful assessment of and for learning (Hughes, 2008). This study expands the literature about prospective teachers' content knowledge, in particular, regarding the combination with MATH taxonomy. The results of the study contribute to understanding the opportunities of continuous assessment in university mathematics education.

	MATH category	*Descriptors of required abilities*
Group A	1. Factual knowledge	Recall previously learned information
	2. Comprehension	Decide the adequacy of a simple definition, interpret and substitute into a formula, recognise examples and counterexamples
	3. Routine procedures	Use procedures in a familiar context beyond factual recall
Group B	1. Information transfer	Decide adequacy of a conceptual definition, apply a formula in a different context, summarize in non-technical terms, explain relationships between objects, etc.
	2. Application in a new situation	Model real-life settings, extrapolate known information to new situations, etc.
Group C	1. Justifying and interpreting	Recognise the limitations in a model and unstated assumptions, discuss the significance of examples and counterexamples, etc.
	2. Implications, conjectures and comparison	Make inductive or heuristic argumentation, prove by rigorous methods, deduce the implications of a given result, construct examples and counterexamples, etc.
	3. Evaluation	Judge the material for a given purpose based on definite criteria

Table 1: Categories in the MATH taxonomy with corresponding descriptors from Smith et al. (1996)

We aimed to evaluate students' work in online tests to gain more understanding for composing supportive and effective continuous assessments in a blended learning environment for mathematics education of prospective primary school teachers. For that purpose, we state the following research questions:

RQ 1: How can students' work in the continuous online assessment be described? What were their achievements in the assessments compared to formal assessments?

RQ 2: How did their work differ concerning the mathematical knowledge at stake and requirements in the questions from the online assessment?

METHOD

Context of the study

This didactic of mathematics course is obligatory for third-year students of teacher studies at our institution. It is allotted two hours of weekly lectures with a cohort of approximately 60 students. The study took place during eight weeks in the second semester during the ERT due to the COVID-19 pandemic. We utilized Moodle for archiving and disseminating lecture notes, literature and other digital materials, synchronous communication through integrated video conferencing tool and live chat, asynchronous communication through forums and direct messages, and online assessments with HTML-based tests. During this time the course content covered scientific methods in mathematics education, that is, induction and deduction, analyses and synthesis, analogy, generalization and specialization, abstraction, and concretization (Kurnik, 2008). The lectures included a definition and description of each method, an explanation of its advantages and limitations and examples of its use in mathematics and mathematics education. This reflected both content and pedagogical aspects of teachers' knowledge.

The obligatory online assessments were in the form of Moodle tests with questions given in advance. Students had a week to complete them. We examined and evaluated their answers and provided feedback. Students' work in the tests was not a part of the formal assessment in the course. Both formal assessments were pen and paper exams, one before the ERT at the end of the first semester and the second under given epidemiological measures at the end of the second semester of the course.

Participants in this study were 60 students in their third year of university studies for primary school teachers during didactics of mathematics.

Study instrument

Moodle tests contained questions in four different mathematical topics from the course content – inductive reasoning in algebraic and geometric context (Ir1-8), mathematical analogy in algebraic and geometric context (An9-12), method of analysis and synthesis in algebraic and geometric context (As13-18), and the area-perimeter problems in a mathematical and real-life context (Ip19-22 in Appendix). Questions were open-ended and related to different aspects of content knowledge and with different requirements. While Moodle tests allow using a variety of question types, closed questions evaluate the correctness of answers, while open-ended questions allow evaluating the whole solution process (Jamil et al., 2022). Students were required to elaborate their solutions and reasoning in writing which promotes mathematical understanding (Kuzle, 2013).

The SMK and MATH category of the questions from Moodle tests are given in Table 2. The placements in categories are based on theoretical consideration regarding literature review and elaborated in Appendix.

SMK categories	*MATH taxonomy categories*					
	A1	*A2*	*B1*	*B2*	*C1*	*C2*
CCK	Ir1, Ir7	Ir2, Ir8	As13	An11, As14		
SCK			An9, Ip19	Ip20, Ip21		Ir3, Ir5
HCK			An10		Ir4, Ir6, As15, As17, Ip22	An12, As16, As18

Table 2: Questions from the Moodle tests regarding SMK and MATH taxonomy categories

Data analysis

The data in this study consisted of students' answers collected from the Moodle tests as obligatory online assessments and their achievement scores in two formal assessments. We performed a qualitative content analysis of students' answers. It is a step-by-step coding procedure where well-defined categories are assigned to each unit of analysis in several cycles (Kuckartz, 2019; Mayring, 2015). We present our analysis according to steps suggested by Kuckartz (2019) with examples of resulting categories.

Step 1: Preparing the data

We exported students' answers from the Moodle tests into xls tables with the textual format. The files students additionally uploaded were downloaded and labelled with the student name and question number. The text from the files was typed and figures were briefly described in the corresponding cell of the xls tables. In some cases, we were unable to access students' answers, because they did not upload a file or they copied a broken link to a file.

The data prepared for content analysis was a matrix with columns corresponding to each question from Moodle tests and rows corresponding to each student's answers.

Step 2: Forming main categories and units of analysis

The unit of analysis was each student's answer to each particular question. Students' answers to a mathematical task can be judged by the appropriateness and correctness of the solution. We decided on the structuring procedure by assessing the units with predetermined ordinal categories (Mayring, 2015): 2 assigned for a correct answer, 1 assigned for a partially correct answer, with an appropriate idea but errors in the solution, and 0 assigned for an inappropriate idea, hence also an incorrect solution.

Since the questions from the tests were open, we expected students' answers to vary in presentation and approach to the solution. We decided on the reductive, summarizing procedure by assigning descriptors of the unit that justify the assigned ordinal category, appropriateness and correctness of the answer, and characterise the solution.

Step 3: Coding data with the main categories

We chose to analyse the data by questions. One researcher, the coder, worked through the units coding with predetermined ordinal categories and assigning descriptors (Figure 1).

Student's answer to Ir2

Broj prirodnih brojeva djeljivih s 2 među prvih 10 prirodnih brojeva.

1:2= 0.5, **2:2= 1,** 3:2= 1.5, **4:2= 2,** 5:2= 2.5, **6:2= 3,** 7:2= 3.5, **8:2= 4 ,** 9:2= 4.5, **10:2= 5**

Broj prirodnih brojeva koji su djeljivi s 2 među prvih 10 prirodnih brojeva su: **2 , 4 , 6 , 8 i 10**

Ordinal category
1 - partially correct
Descriptors
Vague mathematical statement
Calculations $a:d$, $a\leq n$.
Listing numbers divisible by d among first n numbers

Note. We translated the text: •*Number of integers divisible by 2 among the first 10 integers. •1:2=0.5 … 10:2=5 •The number of integers divisible by 2 among the first 10 integers are: 2, 4, 6, 8 and 10.*

Figure 1: Example of student's answer in question Ir2 and coding procedure in Step 3

Step 4: Forming and revising categories and final working through material

The reliability of a qualitative content analysis depends on the categories described for the coding procedure (Kuckartz, 2019). After the first working through units, the two researchers discussed the ordinal categories and descriptors retrieved in the coding process. Researchers made a settling agreement about ordinal categories in each question. They used the descriptors to create data-driven categories, that is inductive category formation, in several cycles until saturation occurred (Kuckartz, 2019; Mayring, 2015). As a final coding procedure, researchers organised, labelled and described ordinal and qualitative categories that cover all instances of students' answers (Table 3). In this way, researchers consensually constructed detailed and precise coding categories and one coder made a final working through units.

Label[1]	*Ordinal category*	*Descriptors of the qualitative category*
P1	2	The volume of an upright three-sided prism with a right triangle base Proof of the true statement
P4	1	The volume of an upright three-sided prism with a right triangle base "The statement is true because it is an analogue"
P5	0	No spatial analogue, reference to the area of a right triangle
L1	1	Spatial analogues of the right triangle copied from literature
I1	1	The volume of a prism imprecisely named as "right three-sided prism"

Note. [1] Qualitative category are characterised and labelled by the origin of students' work in three ways as literature (L), peer (P) or individually (I) oriented work.

Table 3: Examples of several categories in question An12 in the final coding procedure in Step 4

Step 5: Category-based analyses and presenting results

The methodology of this study was grounded on qualitative content analysis. Categorising students' answers allowed for quantitative analysis regarding students' achievement and frequencies of answers. We used descriptive statistics to analyse the data obtained from content analysis. The nature of ordinal categories was fitting to analyse students' achievements. The values 0, 1 and 2 from the ordinal categories in questions from Moodle tests were used as the cumulative scores in online assessment. The qualitative categories and their frequency were used to analyse students' work concerning MATH categories and the content of questions from the Moodle tests.

RESULTS

Students' work and achievement in assessments

The test interface in the Moodle platform allows typing text, including mathematical typesetting, and inserting figures into the answer field. Students often (in particular in questions As14-18, Ip20-21) skipped these options and separately uploaded photographs of their pen and paper work (Figure 3). This meant additional work for lecturers, with downloading and viewing files in a desktop programme, compared to viewing, evaluating, and providing feedback within the Moodle test environment.

The first working through units in Step 2 of the analysis suggested that students' answers could be characterised by the origin of their work in three ways:

- A student copied the excerpts from the lecture notes or literature in their answer to a question – this is *literature-oriented work (L)*. Such categories were easily recognised since students retyped the text word to word or inserted the screenshot of the original material (see Figure 2).
- Two or more students wrote the same answer to a question – this is *peer-oriented work (P)*. The peer-oriented categories were inductively formed as described in Step 4 of the content analysis procedure. We opted that students' answers were assigned in the same peer-oriented category if the answers were identical (see Figure 3 and Figure 4).

– A student had an original answer to a question – this is *individually oriented work (I)*.

A student typed the solution

U 17. stoljeću je njemački matematičar G. W. Leibniz dokazao da je za svaki pozitivan broj broj n^3 - n djeljiv s 3, broj n^5 - n djeljiv s 5, broj n^7 - n djeljiv sa 7. Na osnovi toga izrekao je hipotezu da je za svaki neparni k i svaki pozitivni broj n i broj n^k- n djeljiv s k. No uskoro je i sam uočio da broj 2^9 - 2 =510 nije djeljiv s 9.

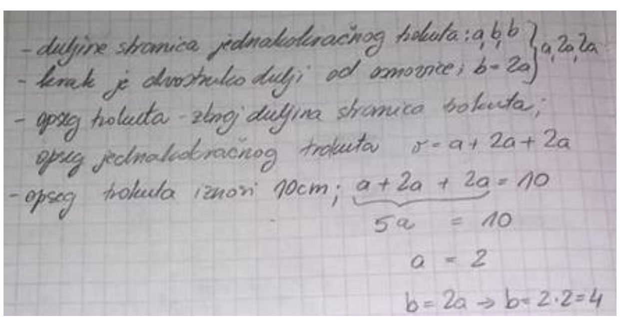
Excerpt from a web document

Primjer 5. U 17. stoljeću je njemački matematičar G. W. Leibniz dokazao da je za svaki pozitivni broj n broj n^3-n djeljiv s 3, broj n^5-n djeljiv s 5, broj n^7-n djeljiv sa 7. Na osnovi toga izrekao je hipotezu da je za svaki neparni k i svaki pozitivni broj n i broj n^k-n djeljiv s k. No uskoro je i sam uočio da broj $2^9-2=510$ nije djeljiv s 9.

Note. We translated identical texts: *In the 17th century German mathematician G. W. Leibnitz proved that for any positive number n, the number n^3-n is divisible by 3, the number n^5-n is divisible by 5, the number n^7-n is divisible by 7. Based on those he stated a hypothesis that for any odd n and every positive number n, the number n^k-n is divisible by k. Soon he noticed that the number $2^9-2=510$ is not divisible by 9.* The text on the right-hand side is an excerpt from the web source *Princip potpune indukcije [The principle of complete induction]* (n.d.). Element. https://element.hr/wp-content/uploads/2020/06/unutra-15008.pdf

Figure 2: Student's input coded with the literature-oriented qualitative category in question Ir8

A student uploaded a photograph of the solution


- duljine stranica jednakokračnog trokuta: a,b,b } a,2a,2a
- krak je dvostruko dulji od osnovice; b=2a
- opseg trokuta - zbroj duljina stranica trokuta,
opseg jednakokračnog trokuta o=a+2a+2a
- opseg trokuta iznosi 10cm; a+2a+2a=10
5a = 10
a = 2
b=2a → b=2·2=4

A student typed the solution

- Duljine stranica jednakokračnog trokuta su a, b, b
- Krak traženog trokuta je dvostruko dulji od osnovice; b=2a
- Iz tog dobijemo da su duljine stranica traženog trokuta a, 2a, 2a
- Opseg trokuta je zbroj duljina stranica trokuta, a opseg traženog jednakokračnog trokuta je O=a+2a+2a
- Računamo: O= 10 cm
  a+2a+2a=10 cm
  5a=10 cm
  a=2 cm
  b=2a
  b=2·2 cm
  b=4 cm
- Traženi trokut je jednakokračan trokut s osnovicom duljine 2 cm i s krakovima duljine 4 cm.

Note. We translated identical texts: •*Lengths of the sides of an isosceles triangle are a, b, b •The leg of the triangle is twice the length of its base; b=2a •Therefore, the lengths of the sides of the triangle are a, 2a, 2a •The perimeter of a triangle is the sum of the lengths of its sides, and perimeter of an isosceles triangle is o=a+2a+2a •We calculate o=10 cm … b=4 cm •The triangle is an isosceles triangle with base length 2 cm and legs length 4 cm.*

Figure 3: Students' inputs coded with the same ordinal and qualitative category in question As15

A student typed the solution coded with P1 in qualitative categories	A student typed the solution coded with P1 in qualitative categories
Ispod su ispisani brojevi koji odgovaraju podacima a=1 cm, b=23 cm, P=23cm^2. "Below are written numbers that fit the data"	a= 1 cm, b= 23 cm, P= 23 cm^2
a=2 cm, b=22 cm, P=44cm^2.	a= 2 cm, b= 22 cm, P= 44 cm^2
a=3 cm, b=21 cm, P=63cm^2.	a= 3 cm, b= 21 cm, P= 68 cm^2
a=4 cm, b=20 cm, P=80cm^2	a= 4 cm, b= 20 cm, P= 80 cm^2
a=5cm, b=19 cm, P=95cm^2	a= 5 cm, b= 19 cm, P= 95 cm^2
a=6 cm, b=18 cm, P=108cm^2	a= 6 cm, b= 18 cm, P= 108 cm^2
a=7 cm, b=17 cm, P=119cm^2	a= 7 cm, b= 17 cm, P= 119 cm^2
a=8 cm, b=16 cm, P=128cm^2	a= 8 cm, b= 16 cm, P= 128 cm^2
a=9 cm, b=15 cm, P=135cm^2	a= 9 cm, b= 15 cm, P= 135 cm^2
a=10 cm, b=14 cm, P=140cm^2	a= 10 cm, b= 14 cm, P= 140 cm^2
a=11 cm, b=13 cm, P=143cm^2	a= 11 cm, b= 13 cm, P= 143 cm^2
a=12 cm, b=12 cm, P=144cm^2	a= 12 cm, b= 12 cm, P= 144 cm^2
a=13 cm, b=11 cm, P=143cm^2	a= 13 cm, b= 11 cm, P= 143 cm^2
a=14 cm, b=10 cm, P=140cm^2	a= 14 cm, b= 10 cm, P= 140 cm^2

Figure 4: Students' inputs coded with the same peer-oriented qualitative category in question Ip19

Each qualitative category was enumerated according to the type of work with a reference to the ordinal category. For example, in Table 3, P4 stands for the fourth among peer-oriented qualitative categories and the answer is partially correct. If a student made additional errors in their peer-oriented work, we assigned them to the inherent qualitative category and corresponding ordinal category (Figure 5).

A student typed the solution coded with P1 in qualitative categories and 2 in ordinal categories	A student typed the solution coded with P1 in qualitative categories and 1 in ordinal categories
KVADRAT	KVADRAT:
pravilan pravokutnik	-pravilan pravokutnik
nasuprotne stranice su paralelne	-susjedne stranice okomite
nasuprotne stranice su sukladne	-susjedne stranice sukladne
susjedne stranice su okomite	-nasuprotne stranice paralelne
susjedne stranice su sukladne	-nasuprotne stranice sukladne
KOCKA	KOCKA:
pravilan kvadar	-susjedne strane okomite
nasuprotne strane paralelne	-susjedne strane sukladne
nasuprotne strane sukladne	-nasuprotne strane paralelne
susjedne strane okomite	-nasuprotne strane sukladne
susjedna strane sukladne	

Note. We translated identical texts (except for one circled sentence): •*SQUARE* •*regular rectangle* •*opposite sides are parallel* •*opposite sides are congruent* •*adjacent sides are perpendicular* •*adjacent sides are congruent* •*CUBE* •*regular cuboid* •*opposite sides are parallel* •*opposite sides are congruent* •*adjacent sides are perpendicular* •*adjacent sides are congruent*

Figure 5: Students' inputs coded with the same qualitative and different ordinal categories in An10

The median scores in the formal assessment before ERT, online assessment and formal assessment after ERT were 60%, 67% and 46%, respectively. We compared students' achievement in the formal assessments and the online assessment from their distribution in the quartiles by their scores in each assessment (Table 4). Though students scored lower in the second formal assessment, they mainly scored in the same quartile of two formal assessments (18 students did not participate, 18 students scored in the same, 13 students scored in the lower, 11 students scored in the higher quartile of second assessment). Students scored higher in the online assessment than in formal assessments, in particular those students who did not participate and those who scored lower in the formal assessments. Students' scores in the formal assessment after ERT were slightly more aligned with their scores in the online assessment (14 students were in the same quartile) compared to their scores in the formal assessment before ERT (10 students were in the same quartile).

Quartiles in online assessment	*Quartiles in the formal assessment before ERT and after ERT*[1]														NP[2]	Total
	1st	2nd	1st	2nd	1st	3rd	2nd	1st	4th	3rd	2nd	4th	3rd	4th		
	1st	1st	2nd	2nd	3rd	2nd	3rd	4th	2nd	3rd	4th	3rd	4th	4th		
1st	2	3		1	1						1				10	18
2nd	2					1			1	2		1		1	7	15
3rd	2			2		1	2		1	1		1	2	4	1	17
4th	1	1	1		1	1		1	1		1	1	1			10
Total	7	4	1	3	2	3	2	1	3	3	2	3	3	5	18	60

Notes. [1] The ordinals in the upper row correspond to the quartile in the formal assessment before ERT and ordinals in the lower row correspond to the quartile in the formal assessment after ERT. [2] Label NP refers to students who did not participate in either of two formal assessments.

Table 4: Distribution of students regarding achievement in the formal and online assessments

Majority of students dominantly used peer-oriented work (Table 5). Almost all students who scored lower in the formal than the online assessment dominantly used peer-oriented work in online assessment. Students who dominantly or evenly used individually oriented work in the online assessment were mainly in the higher quartiles in the formal assessments. Inspecting for qualitative categories showed that students from lower quartiles in both formal assessments who dominantly used peer-oriented work mainly provided the same answer as a student from higher quartile in either of the two formal assessments.

Dominant origin of work	*Achievements in the formal assessments compared to online assessment*[1]										*Total*
	Both lower			*Same or one lower*				*One higher*			
	2nd	*3rd*	*4th*	*1st*	*2nd*	*3rd*	*4th*	*1st*	*2nd*	*3rd*	
Peer	7	5	5	8	1	3	4	7	3	4	47
Individual				1			1		3	2	6
Mixed	1			1		1		1		2	7
Total	8	5	5	10	1	4	5	8	6	8	60

Notes. [1] Label in the upper row suggests students scored in lower or higher quartile in the formal assessments than the quartile in online assessment marked with ordinal in the lower row.

Table 5: Distribution of students regarding compared achievement in assessments and dominant origin of their work

Students' work concerning content and requirements in questions

Students' answers concerning the type of work (literature-L, peer-P or individual-I oriented) and values (correct-2, partially correct-1 or incorrect-0) varied across questions (Figure 6). The number of qualitative categories with peer-oriented work also varied across questions (Figure 7).

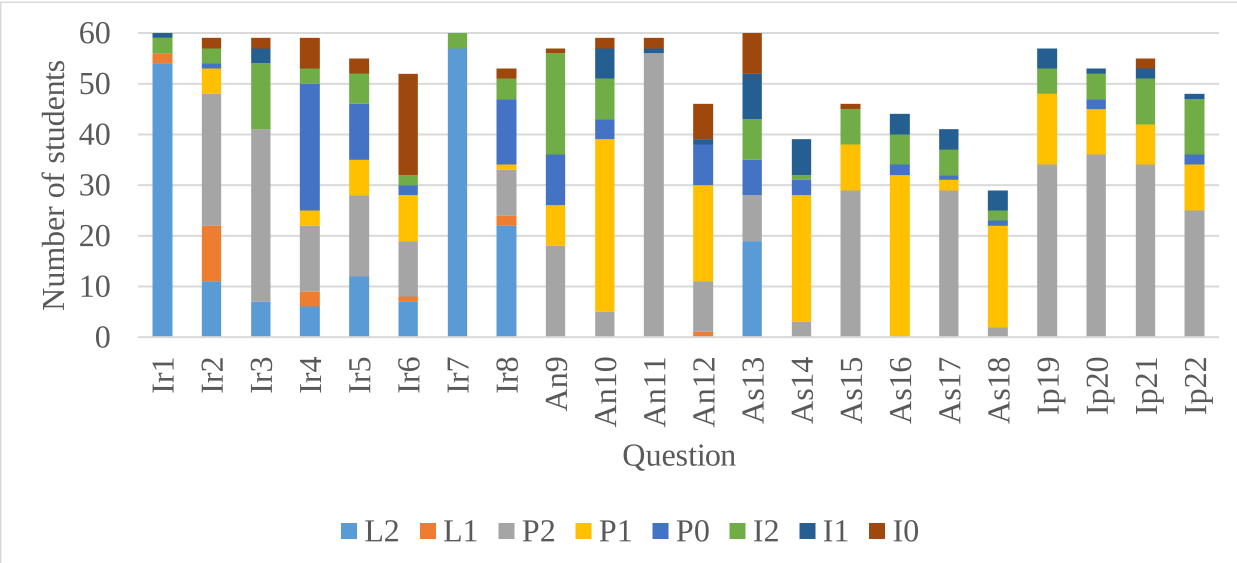


Figure 6: Distribution of students concerning the type of work and value of their answers across questions

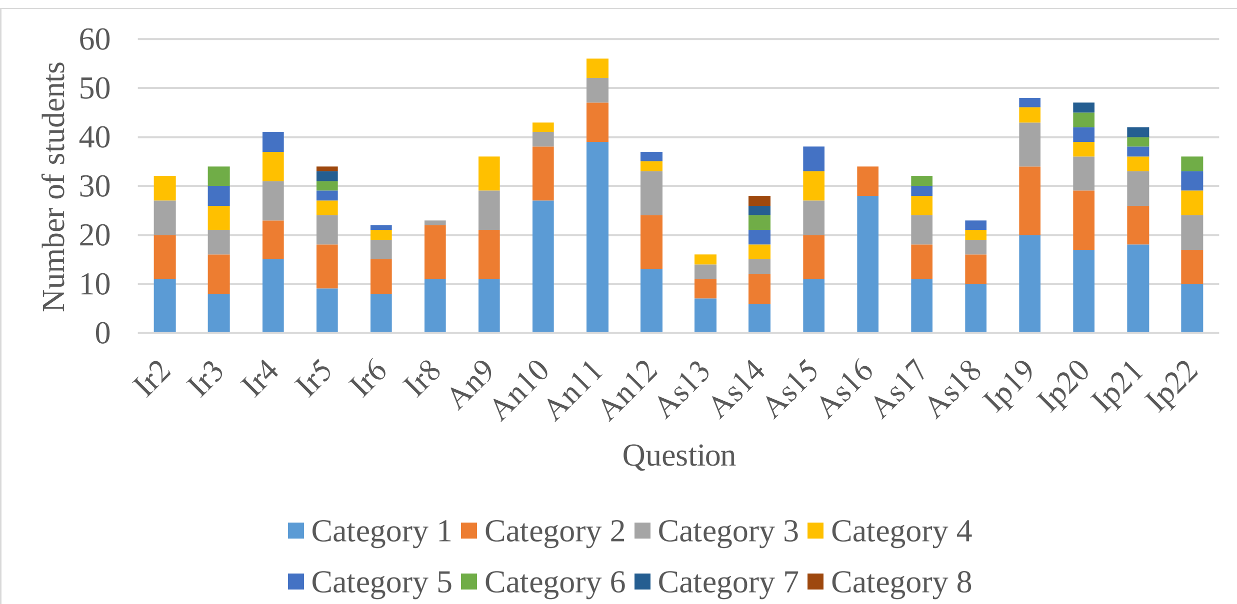


Figure 7: Distribution of students concerning different peer-oriented categories across questions

Questions about inductive reasoning in CCK and A1, Ir1 and Ir7, had almost all literature-oriented correct answers, with students correctly retrieving the definitions from the literature. Questions Ir2 and Ir8 in CCK and A2 had a significant number of literature and peer-oriented answers. In question Ir2, students provided, in a full or partial account, correct examples of complete (finite) induction from literature, and in each of the four peer-oriented categories, a correct example analogous to some examples given in lectures. In question Ir8, students provided correct counterexamples of incomplete (infinite) induction from literature and correct counterexamples in one of the three peer-oriented categories.

Other questions about inductive reasoning (Ir3-6) had a share of literature, and peer and individual-oriented answers. In questions Ir3 and Ir5 categorised in SCK and C2, some students retrieved examples of patterns from literature. Others constructed a variety of analogous and original examples in individual and peer-oriented work, which were mainly appropriate in the numerical infinite context in the former, but not the geometric infinite context in the latter question (Figure 8). In questions Ir4 and Ir6 categorised in HCK and C1, the number of correct answers was smaller than in questions Ir3 and Ir5. Students wrote an incorrect algebraic expression for the general term or vague explanation of the pattern, especially in the case of the non-analogue examples they gave in the preceding questions.

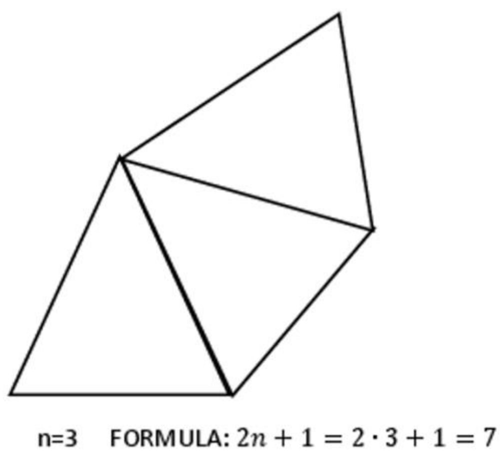


Figure 8: Student’s incorrect example for growing geometric pattern in question Ir5

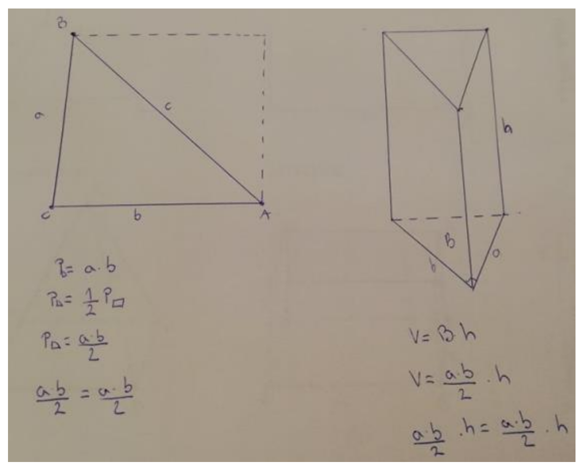


Figure 9: Student’s proof of analogy (P1 qualitative category) in question An10

Questions about analogous geometric objects, An9 in SCK and B1 and An10 in HCK and B1, had dominantly peer-oriented answers with few different categories and a significant number of individually oriented answers. Students’ answers in An9 differed in minor features, the shapes (particularly the pair triangle-tetrahedron), sketching and labelling geometric figures and solids. Students’ answers in An10 differed significantly with a variety of well-observed analogous properties of square and cube. The dominant peer-oriented partially correct answer was focused on one analogues property, that square and cube both have congruent sides. In both questions, students mainly erred in terminology related to the properties of geometric objects.

Question An11 about analogous expression in CCK and B2 had the largest proportion of peer-oriented correct answers. Almost all students made the same correct mathematical analogy of given algebraic equality, and qualitative categories had the same mathematical content with different representations. Question An12 about an analogous statement in geometry in HCK and C2 had two peer-oriented categories with a correct analogy (P1 in Table 3), but only some students provided proof (Figure 9) which counted for a complete, correct answer. Other peer and individual-oriented answers were partially correct due to imprecise statements or missing justification (P4 and I1 in Table 3).

Question As13 about the method of analysis and synthesis in CCK and B1 had differently oriented answers and the largest share of individually oriented answers. Some students retrieved parts of the definition of the method of analysis and synthesis from the literature. Others described the method in their own words, revealing different conceptions. For example, a student's individually oriented partially correct answer was focused on the algorithmic procedure as applicable in question As14, and the dominant peer-oriented answer, "what we do by analysis, we can check by synthesis", did not correctly convey the idea. In question As14 about analysis and synthesis in CCK and B2, we were unable to access all students' answers. Students provided the calculation part of the analytic-synthetic procedure of proving an algebraic inequality. Their answers differed in the order of algebraic manipulation, and they were partially correct due to omitting some elements in the overall procedure.

In the geometric construction problems, we were unable to access all students' answers. The answers were dominantly peer-oriented, mainly correct in questions As15 and As17 in HCK and C1, and partially correct in questions As16 and As18 in HCK and C2 category. In the former, students followed the correct idea in the analysis of the geometric problem and erred in terminology and calculation. In the latter, when justifying the construction, students mainly focused on the dominant property of the geometric figure, that is, the type of the triangle or the perimeter of the triangles. The larger number of categories in questions As17 and As18 than As15 and As16 did not come from students' work with properties of the triangles but their use of different measuring units.

Questions about area-perimeter problems (Ip19-22) mainly had peer-oriented correct answers. The difference between questions was in the variety of answers. Question Ip19 in SCK and B1 had fewer different individual and peer-oriented answers, with students appropriately using the formula and systematically organizing the data similar to the example given in lectures. In questions Ip20 and Ip21, both in SCK and B2, students approached solving the problems in a real-life context in different ways, by solving equations, drawing, tabulating outcomes or calculating values (Figure 10). In question Ip22 in HCK and C1, there was a variety of peer and individual-oriented answers. Students had different focuses when discussing the limitation of the task in question Ip19 – providing examples and non-examples, calculating the values, discussing the properties of figures, and applying mathematical statements or reasoning intuitively. Following are examples of students' answers from different categories.

P1: „If it is an even number not divisible by 4, the lengths of the sides of the rectangle with the maximum area **cannot be calculated** directly. If it is an odd number, we cannot use **integers**."

P2: „ If it is an even number not divisible by 4, the solution is not an **integer**. Eg., o=50 cm, P=12.5·12.5=156.25 cm^2. The student would investigate sides with **integer** lengths and conclude the maximum area is 156. If it is an odd number, the side lengths are not **integers**. If a and b are integers, then $o=2(a+b)$ is divisible by 2! This task is **not appropriate** for students in primary education."

P3: „ If it is an even number not divisible by 4, the lengths of the sides of the rectangle with the maximum area **cannot be calculated** directly (SQUARE). If it is an odd number, for a rectangle with the maximum area we cannot use **integers** but **real numbers**."

P4: „ If it is an even number not divisible by 4, the solution is **not a square but a rectangle** with approximate side lengths. If it is an odd number when investigating for a rectangle with the maximum area the resulting side lengths are real numbers."

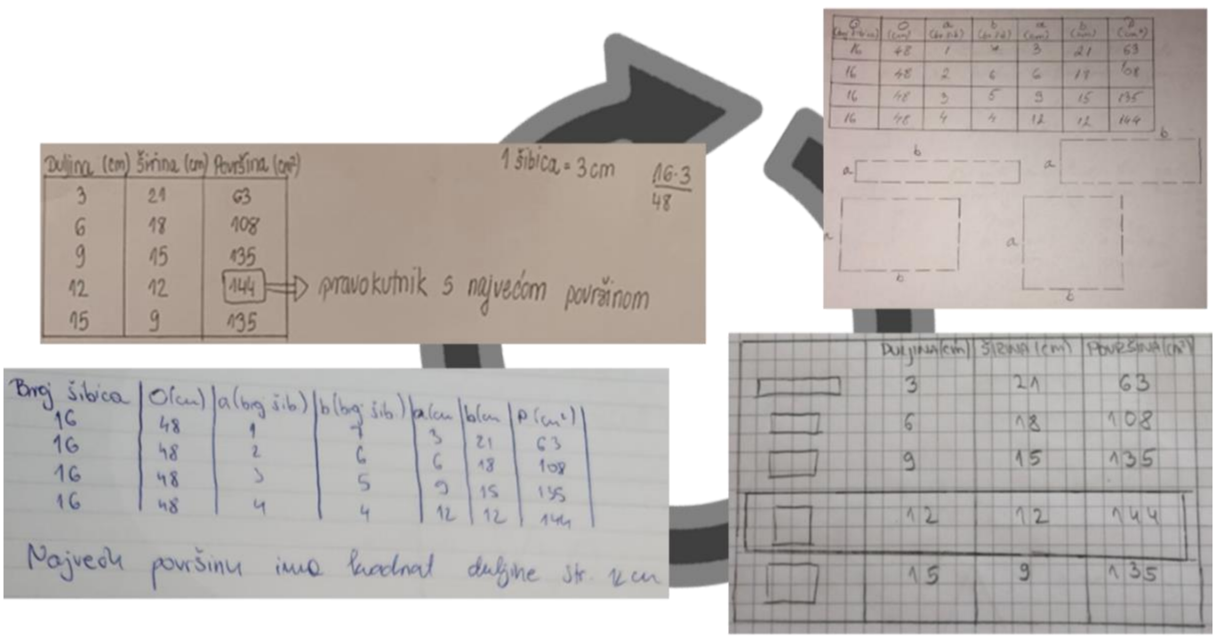


Figure 10: Students' different representations within different qualitative categories in question Ip21

DISCUSSION

During the ERT, students of primary teacher studies in the course of didactics of mathematics worked in continuous, obligatory, non-graded, online assessments in the form of Moodle tests. The purpose of the assessments was to engage students in continuous independent work and provide them with feedback; they would reflect on the course content and develop from feedback. Though the goals of the course work were aligned with the idea of effective continuous assessment there was no evidence that the assessments affected students' achievement in the course. Overall lower achievement in the second formal assessment could be due to different course content, the unprecedented circumstances of the ERT or the lack of preparation. We found that students approached the online assessments in different ways; they relied on literature, and peer or individual-oriented work. Students who scored higher in either of the two formal assessments were more engaged in individual work in the online assessments. The peer-oriented work could have been produced in two ways – as a collaborative student's work or as a copied work from one engaged student. The majority of students worked peer-oriented and their answers were mainly equal to a high achieving student's answers. Thus, high achieving students appeared as managers of peer work, and other students appeared to have invested minimal effort in the online assessments. In the context of study approaches, most students had a surface-strategic learning approach, which was to complete the assignment by submitting the best possible answers and relying on peer-oriented work. Students with a deep learning approach submitted original answers, not necessarily correct since they invested time and effort in individual work. Their approach was aligned with the purpose of the exercises. Reflecting on the results in general, though students completed the frequent assessments which balanced their workload, they did not engage in regular, independent, active work that would be a prerequisite of a deep approach. Focusing on students' work in particular questions, we gained information on choosing appropriate and interesting tasks with supportive feedback to increase students' interest, motivation, and engagement, directing them towards a deep approach to learning.

Questions from the Moodle tests could be judged for their fitness for online assessment and continuous assessment. Evaluating open questions in the Moodle environment, especially files uploaded rather than embedded, was time-consuming, compared to the automated evaluation of closed questions in Moodle tests or evaluating pen and paper assessments. Some of the open questions could be rearranged into closed questions without loss in the requirements and with corresponding, pre-defined feedback (Jamil et al., 2022). For example, questions Ir1 and Ir7, that was recalling the definition, could be \`select missing words´ question type, question An9, which was deciding about analogue objects, could be \`matching´ question type, and questions An11 and As14, that was extrapolating known information to different situation, could be \`multiple choice´ question type to select correct expression and \`drag and drop´ to arrange steps of the procedure correctly. Open, \`essay type´ questions that were easy to evaluate in Moodle environment were questions Ir2 and Ir8, questions Ir3-6, that were about retrieving or constructing examples and counterexamples, questions An10 and As13, that were explaining the relationship between objects, and summarizing mathematical discourse in non-mathematical terms, and question Ip22, that was discussing limitations of a mathematical task. Feedback in these questions could be criterion-referenced feedback valued against a set of pre-defined requirements and additionally, by the provision of content analysis, general reflective feedback could be constructed for dissemination and discussion. Other questions, solving geometric construction problems, making and proving conjectures and modelling real-life problems seemed less appropriate for the online environment and independent work.

Questions in formal and informal assessments should appropriately reflect the course content and be of different types with varying degrees of difficulty (Hughes, 2008; Korhonen et al., 2015). The subjective difficulty of questions could be assessed from the frequencies of ordinal categories and the diversity of qualitative categories in each question. Questions with almost all correct answers, dominant literature-oriented answers or one dominant qualitative category are unsuitable for differentiation, constructive feedback or promoting a deep approach. Questions with almost all partially correct or incorrect answers are also unsuitable either for assessing prospective primary school teachers' knowledge or for this type of assessment. Questions which engaged students in work were those with varied ordinal categories which were also challenging for students and those with varied qualitative categories which were productive for students.

There were differences in students' engagement in questions in different SMK and MATH taxonomy categories (Figure 11). Students seemed least engaged when reproducing formal definitions of a mathematical notion (Ir1,7 in CCK and A1). But they seemed more engaged when required to propose examples for the same notions (Ir2,8 also in CCK but in A2), or to reflect on the definition of a mathematical notion (As13 also in CCK but in B1). When transferring mathematical knowledge, tasks set in a real-life context appropriate for primary education (Ip20,21 in SCK and B2) appeared more engaging than tasks set in a purely mathematical context (An11, As14 in CCK and also in B2). In the former, students used various representations in their solution, and in the latter, they used presupposed form of the solution. Working with primary level notions and applying primary level formulas (An9 and Ip19 in SCK and B1) was not as productive as constructing examples of numeric and geometric patterns appropriate for primary education (Ir3,5 also in SCK but C2) nor as challenging as describing general, mathematical properties of a primary level notion (An10 in HCK and also in B1). Similarly, discussing the conditions and limitations of a primary level task seemed challenging and productive for students (Ip22 in HCK and C1). Solving geometric construction tasks appropriate for primary mathematics education using the method of analysis and synthesis did not appear productive for students (As15-18 in HCK and C1 or C2). In particular, students recognized the solution in the context of primary education but were not able to communicate the solution in the context of formal mathematics. For students, the most challenging seemed to be generalizing, deducing, and justifying formally about primary level content (Ir4,6 in HCK and C1, An12 in HCK and C2). Corresponding questions had a larger proportion of partially correct or incorrect answers compared to the questions with similar content and different requirements. In particular, students had more difficulties with questions in geometric and infinite contexts than questions with algebraic and finite contexts.

Unsuitable	Less engaging	Productive	Engaging	Challenging	Unsuitable
CCK A1: Ir1, Ir7 CCK B2: An11, As14	SCK B1: An9, Ip19	CCK A2: Ir2, Ir8 CCK B1: As13 SCK B2: Ip20, Ip21	SCK C2: Ir3, Ir5 HCK B1: An10 HCK C1: Ip22	HCK C1: Ir4, Ir6 HCK C2: An12	HCK C1: As15, As17 HCK C2: As16, As18

Figure 11: Distribution of questions regarding suitability and engagement

Questions with a larger proportion of individually oriented answers and multiple categories of peer-oriented work are preferable. Students' answers to such questions provide a variety of discourses, examples, or strategies that can be shared and discussed among peers thus enriching their communication, conceptions and strategies related to mathematical notions. Such are the following types of questions:

Type 1: Retrieving (Ir2 and Ir8) and creating examples (Ir3 and Ir5) for mathematical and primary level notions,

Type 2: Describing, discussing, and making judgments, in particular, summarizing in non-mathematical terms a definition of a mathematical notion (As13), comparing mathematical properties of primary level objects (An10), making and proving mathematical conjectures about primary level objects (An12), and discussing mathematically primary level problems (Ip22),

Type 3: Solving contextual problems related to a particular primary level problem (Ip20-21).

Type 1 and 2 questions gave insight into students' (mis)conceptions about mathematical notions, that is, their concept image as a whole collection of ideas, representations, examples, and relations, formed mentally about the notion (Tall & Vinner, 1981). Such questions are essential to learning and understanding a mathematical notion by developing a comprehensive and suitable concept image (see Horzum & Ertekin (2018), Ulusoy (2021), Vinner (1991)). Type 2 and 3 questions revealed different strategies and focus in students' mathematical work, their mathematical thinking style as a preferred way of understanding, presenting and thinking about mathematical notions (Borromeo Ferri, 2010) or using and connecting different representations as an indicator of specialized content knowledge (Steele, 2013). The visual or analytic style was reflected in discussing geometric or measurable properties of figures, and the problem-solving approach used in the contextual problem (Figure 10). For example, students drew figures - visual thinking style or calculated different outcomes - analytical thinking style, to answer the question.

Limitations, implications, and further research

The participants of the study and the content of the questions were limited by our particular context. Though the results of our study are not generalized, they contribute to practice and research by considering the categories of assessment questions. Students' work agreed with the requirements in the assigned MATH category. Questions with different content in our study were assigned to a few categories in the MATH taxonomy. Any assessment should strive to contain questions across all mentioned categories. More information on students' work in questions from different categories would additionally support the discussion about appropriate types of questions for continuous online assessment. In the context of our study, that option was dismissed since it would have increased students' workload significantly.

The results of our study were inconclusive about how continuous online assessment affected students' achievements in the course. For one, many students relied heavily on peer-oriented work hence the issues of supervision and plagiarism arose, and their cumulative achievement calculated from the ordinal category assigned to their answer might not be their achievement. Second, the underlying idea of the online assessments was that students would prosper from the feedback regardless of the correctness of their answers hence students' lower achievement in online assessments need not imply lower achievement in the formal assessment. Questionnaires and interviews with students about their knowledge, experience and attitude could provide additional insight into their work in online continuous assessment.

The course content limited the nature of questions regarding mathematical knowledge for teaching. The questions were inclined toward the formal mathematical knowledge of prospective teachers.

The results of this study implicate that designing assessments for prospective teachers might include two-dimensional choices, reflecting on content knowledge and task requirements. The MATH taxonomy seemed compatible with SMK categories but no conclusions about its compatibility with PCK categories can be made. Steele (2013) also suggested designing tasks to access interactions between different categories of mathematical knowledge for teaching. This is the direction for our further practice and research.

CONCLUSION

Online assessments in the form of Moodle test implemented in our study provided information about prospective teachers' engagement in continuous work, designing online tests in mathematics education, choosing questions for continuous assessment, and some issues in students' mathematics knowledge. The methodology used in this study can be adapted to different contexts. Content analysis of students' answers proved an invaluable tool, in particular, by forming the qualitative categories which described the origins, correctness and characteristics of students' answers. Categorizing questions regarding mathematics knowledge for teaching and MATH taxonomy seemed fitting and useful for assessing prospective mathematics teachers, both for achieving the goal of the study and for implementing it in our teaching practice.

The results of our study are aligned with the study by Lebeničnik et al. (2015) who found that future teachers are more inclined to passively receive information than actively engage and collaborate on educational tasks and Tanujaya et al. (2021) who reported inactive collaboration and copying other students' solution as issues with online learning. However, continuous, non-graded, online assessments with engaging tasks which complement the regular lectures in a blended learning environment might be such activities that promote deep learning. Students participate without pressure to obtain a particular grade and they learn from the teacher's feedback, while the teacher reorganizes the teaching and discusses different answers and approaches. This kind of work is very demanding and time-consuming hence it is important to thoughtfully formulate questions and choose the format of the assignment.

The results of our study suggest that the questions that require creating examples, discussing definitions and properties, and solving contextual problems prompted students' active engagement and provided insight into students' conceptions and thinking styles founding for rich and constructive feedback. It was the case for both mathematical and primary level content. These types of tasks can be included in frequent, individual assignments with formative, individual feedback and reflective, comprehensive feedback on examples and non-examples of solutions students provided. The tasks that required argumentation or generalization about primary level content from ab advanced point of view (horizon knowledge) were challenging for students. Such questions can be implemented as occasional, intensive, group work with peer evaluation.

Students approached asynchronous online tests strategically and worked with peers to complete them. In that light, more effective continuous assignments can be designed in two ways, as easy-to-evaluate independent, engaging assessments or as collaborative activities. Each of these elements could be incorporated as a blended learning environment to engage students in continuous work.

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APPENDIX

Questions from Online Assessments

	Question from the online assessment	*Requirements from MATH taxonomy category*	*SMK domain*
Ir1	Explain briefly complete induction.	A1 Recall the definition of complete induction	CCK includes knowledge of complete induction as mathematical content
Ir2	Provide an example for complete induction.	A2 Recognize an example for complete induction (finite context)	CCK includes knowledge of complete induction as mathematical content
Ir3	Create a rule for a sequence of integers. Write several first terms in the sequence.	C2 Construct an example of an infinite integer sequence	SCK includes constructing examples for integer sequence as school content
Ir4	Describe the rule for the sequence with words and the *n*-th term with symbols.	C1 Discuss the properties of an infinite integer sequence	HCK includes generalization about integer sequence as mathematical work with school content
Ir5	Create a rule for a growing geometric pattern. Draw several first figures by the pattern.	C2 Construct an example of a growing geometric pattern	SCK includes constructing examples of geometric pattern as school content
Ir6	Describe the rule for the growing geometric pattern with words and the value for the *n*-th figure with symbols.	C1 Discuss the properties of a growing geometric pattern	HCK includes generalization about geometric pattern as mathematical work with school content
Ir7	Explain incomplete induction.	A1 Recall the definition of incomplete induction	CCK includes knowledge of incomplete induction as mathematical content
Ir8	Find an example of a false claim obtained by incomplete induction.	A2 Recognize counterexample for incomplete induction (infinite context)	CCK includes knowledge of incomplete induction as mathematical content
An9	Write out all the geometric plane figures mentioned in primary mathematics education. For each figure write the name of its spatial analogue. Draw the pairs of analogues objects. Name the vertices and sides of the figures and solids.	B1 Decide about the analogy between corresponding plane figures and solid shapes	SCK includes presenting ideas and selecting representations of geometric objects as school content
An10	Explain why a square and cube are analogues objects.	B1 Explain analogous properties of square and cube	HCK includes argumentation about geometric objects as mathematical work with school content
An11	Write the analogue of the equality $\|ab\| = \|a\|\|b\|$.	B2 Extrapolate known algebraic relation to a different setting by analogy	CCK includes stating analogous algebraic equality as mathematical work

	Question from the online assessment	Requirements from MATH taxonomy category	SMK domain
An12	State the analogue of the claim “The area of a right triangle equals the half of the product of its catheti lengths”. Is the analogous claim true? Explain.	C2 Make a conjecture by stating a spatial analogue of a known result in the planar geometry and formally prove or disprove the conjecture	HCK includes stating analogy for the area of triangle as mathematical work with school content
As13	Explain (in your own words) why analysis and synthesis make a unique method.	B1 Summarize in non-mathematical terms the relationship between analysis and synthesis in the scientific method	CCK includes analysis and synthesis method as mathematical work
As14	Let a and b be positive real numbers. Prove the inequality $\frac{a}{b}+\frac{b}{a}\geq 2$.	B2 Extrapolate known procedure of proving an algebraic inequality by the method of analysis and synthesis to a different setting	CCK includes proving algebraic equality as mathematical work
As15	Construct an isosceles triangle with 10 cm perimeter, and with its legs length equal twice the base length. Analyse the problem.	C1 Recognize and interpret assumptions by analysing geometric construction problem	HCK includes analysing simple construction problem as observing school content from advanced point
As16	Construct an isosceles triangle with 10 cm perimeter, and with its legs length equal twice the base length. Synthesise the solution to the problem.	C2 Deduce solution validity by synthesising the geometric construction	HCK includes synthesizing simple construction problem as observing school content from advanced point
As17	Construct an equilateral triangle with a perimeter equal to the perimeter of an isosceles triangle with 1 dm legs length, and the 85 mm base length. Analyse the problem.	C1 Recognize and interpret assumptions by analysing geometric construction problem	HCK includes analysing simple construction problem as observing school content from advanced point
As18	Construct an equilateral triangle with a perimeter equal to the perimeter of an isosceles triangle with 1 dm legs length, and the 85 mm base length. Synthesise the solution of the problem.	C2 Deduce solution validity by synthesising the geometric construction	HCK includes synthesizing simple construction problem as observing school content from advanced point
Ip19	Explore the rectangles with perimeter *o*=48 cm, and integer length of its sides. Write out all the rectangles. Include sides lengths and area.	B1 Apply formulas for the perimeter and area of a rectangle in a particular context	SCK includes presenting ideas and connecting representations about area/perimeter relation as school content

	Question from the online assessment	*Requirements from MATH taxonomy category*	*SMK domain*
Ip20	George has 144 concrete panels shaped like a square with a 1-meter sides length. He will use it to pave a part of his yard shaped like a rectangle. He will surround that part with an expensive fence. What should be the length and width of the concrete part of the yard so that it requires the least fencing?	B2 Model real-life setting with perimeter and area of a rectangle	SCK includes discussing ideas, selecting representations, interpreting solutions about area/perimeter relation as school content
Ip21	Vita is making rectangles using matches with 3 cm length. She has 16 matches. Which of the rectangles has the largest area?	B2 Model real-life setting with perimeter and area of a rectangle	SCK includes discussing ideas, selecting representations, interpreting solutions about area/perimeter relation as school content
Ip22	Note that the value of the perimeter given in Ip19 is a multiple of 4. What if the value of a given perimeter is an even number that is not a multiple of 4? What if the value of a perimeter is an odd number?	C1 Recognizing the limitations occurring in the solution of a mathematical task related to the area and perimeter of a rectangle by changing the initial values in the task	HCK includes argumentation about conditions in area/perimeter task as mathematical work with school content